MCA DEGREE I SEMESTER EXAMINATION DECEMBER 2013

CAS 2104 DISCRETE MATHEMATICAL STRUCTURES

(Regular & Supplementary-2010 Revision)

Time: 3 Hours | Maximum Marks: 50

PART A

(Answer ***ALL*** questions)

$(15 \times 2 = 30)$

I. (a) Prove the equivalence $(p \wedge q)\ v\left(P \wedge \neg q\right) \equiv p$

(b) Explain universal quantifiers and existential quantifiers.

(c) Define lattice. Give an example.

II. (a) State the counting principle and pigeonhole principle.

(b) Prove that $C(n,r) = C(n-1,r) + C(n-1,r-1))$

(c) Let $A = \{1,2,3,4\}$ and $R = \{(x,y) / x \le y\}$. Write the relation matrix M_R.

III. (a) What is the order and degree of a recurrence relation?

(b) Solve the recurrence relation of the Fibonoci sequence of numbers;

$F_n = F_{n-1} + F_{n-2;}\ n \ge 2$ with the initial conditions $F_0 = F_1 = 1$

(c) Find the number of integer solutions of the equation

$X_1 + X_2 + X_3 + X_4 = 0$; such that $X_i \ge 0, i = 1,2,3,4,$.

IV. (a) State the axioms of Boolean algebra.

(b) Prove the Boolean identity: $x \oplus \left(X^1 * y\right) = x \oplus y$

(c) Minimize the Boolean expression: $x.y' + x'.y + x'.y'$.

V. (a) Explain different types of grammars.

(b) Find the language L(G) generated by the grammar G with variables $\{S, A, B\}$, terminals $\{a, b\}$ and productions $S \to a, S \to Sa, S \to b, S \to bS$

(c) Compare DFA and NFA.

PART B

$(5 \times 4 = 20)$

VI. Prove that the relation congruence modulo 3 given by $R = \{(x,y) / x - y \text{ is divisible by } 3\}$ over the set of integers Z, is an equivalence relation. Determine the equivalence classes generated by the elements of Z. Also find the quotient set Z/R.

OR

VII. Explain the principle of mathematical induction.

Show that $1+1/\sqrt{2}+1/\sqrt{3}+1/\sqrt{4}+.......1/\sqrt{n} > \sqrt{n}$ for $n \geq 2$.

VIII. Let R be the relation on the set $A=\{1,2,3,4\}$ such that the relation matrix M_R is

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find the symmetric and transitive closures of R.

OR

IX. Find the number of integers between 1 and 250 both inclusive that are not divisible by any of the integers 2,3,5 and 7

X A particle is moving in the horizontal direction. The distance it travels in each second is equal to two times the distance it traveled in the previous second. If a_r denotes the position of the particle in the r^{th} second, determine a_r, given that $a_0=3$, $a_3=10$.

OR

XI. Solve the following recurrence relation.

$a_{n+1} - 8a_n + 16a_{n-1} = 4^n;\ n \geq 1;\ a_0 = 1, a_1 = 8.$

XII. Using K-map, simplify the following Boolean expression

$F(x,y,z) = xy^1z + x^1yz + xyz + x^1y^1z + xyz^1$

Draw the logical circuit for the resultant expression.

OR

XIII. In any Boolean algebra, show that

(i) $xy^1 + x^1y = 0$ iff $x = y$

(ii) $(x+y^1)(y+z^1)(z+x^1) = (x^1+y)(y^1+z)(z^1+x)$.

XIV. State and prove pumping lemma. State its applications.

OR

XV. For $\sum=\{a,b\}$ construct DFA's that accept the sets consisting of;

(i) All strings with exactly one 'a'.

(ii) All strings with at least one 'a' and exactly two b's

(iii) All strings with exactly two a's and more than two b's.

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