MCA.II/05.15.0394 Reg. No. [ ]

A

M.C.A. DEGREE II SEMESTER EXAMINATION MAY 2015

CAS 2204 DISCRETE STRUCTURES AND GRAPH THEORY

(Regular)

Time: 3 Hours Maximum Marks: 50

PART A

(Answer *ALL* questions)

$(15 \times 2 = 30)$

I. (a) Prove that $(A \cap B) - C = (A - C) \cap (B - C)$.

(b) Show that the relation 'is congruent modulo 4 to' on the set of integers $\{0,1,2,3...10\}$ is an equivalence relation.

(c) Define ring and when does it become a division ring.

II. (a) State and prove DeMorgan's theorems.

(b) Draw logic diagram to represent the Boolean expression $(xy)' + xyz$

(c) Construct a truth table for the statement $(\sim p \vee q) \wedge \sim r$.

III. (a) Find the number of ways in which 7 different beads can be arranged to form a necklace.

(b) Explain pigeon hole principle with a suitable example.

(c) Find first four terms of the recurrence relation $a_k = a_{k-1} + 3a_{k-2}$, for all integers $k \geq 2,\ a_0 = 1,\ a_1 = 2$.

IV. (a) Show that the maximum number of edges in a simple graph with 'n' vertices is $\frac{n(n-1)}{2}$.

(b) Define a Hamiltonian graph. Give an example.

(c) What is a spanning tree? Explain with an example.

V. (a) Distinguish between k-colouring and chromatic number of a graph.

(b) Explain max flow-min cut theorem.

(c) Define chromatic polynomial of a graph.

PART B

$(5 \times 4 = 20)$

VI. Show that the set S of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, where $a,b \in R$ is a field with respect to matrix addition and matrix multiplication.

OR

VII. In a group of 70 people, 37 like coffee,52 like tea and each person likes at least one of the two drinks. Calculate (i) how many people like both coffee and tea? (ii) how many people like coffee but not tea?

VIII. Find all sub lattices of D_{24} that contain five or more elements.

OR

IX. Draw Karnaugh map and simplify the Boolean expression
$Y = A\overline{B}\overline{C} + AB\overline{C} + ABC + A\overline{B}C + A\overline{B}\overline{C}$

(P.T.O.)

X. Solve the recurrence relation of the Fibonaci sequence of numbers $f_n = f_{n-1} + f_{n-2}$, $n \geq 2$ with the initial condition $f_0 = 1, f_1 = 1$.

OR

XI. Use generating functions to solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n \quad a_0 = 2, a_1 = 1$

XII. Define a tree. Show that a tree of *n* vertices has $n-1$ edges.

OR

XIII. Explain Kruskal's algorithm for minimum spanning tree with a suitable example.

XIV. Explain Ford and Fulkerson algorithm with an example.

OR

XV. Show that an *n* vertex graph is a tree if and only if its chromatic polynomial $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$.

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