MCA.I/12.15.1097 Reg. No. | | | | | | | | |

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MCA DEGREE I SEMESTER EXAMINATION DECEMBER 2015

CAS 2105 DISCRETE STRUCTURES AND GRAPH THEORY

(Regular)

Time: 3 Hours Maximum Marks: 50

PART A

(Answer *ALL* questions)

$(15 \times 2 = 30)$

I. (a) Prove that $A-(A-B) \subseteq B$.

(b) Construct a truth table for the compound proposition $(p \wedge q) \vee \sim r$.

(c) Obtain the principal disjunctive normal form of $q \vee (p \vee \sim q)$.

II. (a) If R and S are equivalence relations on the set A, prove that $R \cap S$ is an equivalence relation.

(b) Explain the digraph of a relation

(c) Find the matrix of the relation R on X relative to the ordering given
$R = \{(1,2),(2,3),(3,4),(4,5)\}$;
ordering of x : 1, 2, 3, 4, 5.

III. (a) Show that the set 'Z' of all positive integers under divisibility relation forms a poset.

(b) Explain Hasse diagram.

(c) Determine whether D_{12} is a finite Boolean algebra or not.

IV. (a) Show that the maximum number of edges in a simple graph with 'n' vertices is $\frac{n(n-1)}{2}$.

(b) Distinguish between k – coloring of a graph and chromatic number of a graph.

(c) Explain Fleury's algorithm with an example.

V. (a) Define an abelian group, with an example.

(b) Explain integral domain.

(c) Let T be the set of even integers. Show that the semigroups $(Z, +)$ and $(T, +)$ are isomorphic.

PART B

$(5 \times 4 = 20)$

VI. Out of 250 candidates who failed in an examination, it was revealed that 128 failed in Mathematics, 87 in Physics and 134 in aggregate. 31 failed in Mathematics and in Physics, 54 failed in the aggregate and in Mathematics, 30 failed in the aggregate and in Physics.

(i) How many candidates failed in all the three subjects?

(ii) How many candidates failed in Mathematics but not in Physics?

OR

VII. Prove that the following propositions are tautology.

(i) $\sim(p \wedge q) \vee q$

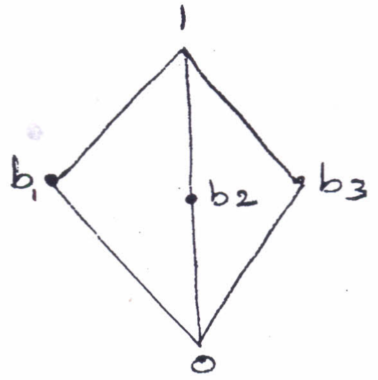
(ii) $p \Rightarrow (p \vee q)$

VIII. Show that the relation 'is congruent modulo 4 to' on the set of integers $\{0, 1, 2, \ldots, 10\}$ is an equivalence relation.

OR

IX. R is a relation on set of integers Z defined by $R = \{(x, y);\ x \in z,\ y \in z,\ (x - y)$ is multiple of $3\}$. Prove that R is an equivalence relation.

X. Prove that the lattice given by the following diagram is modular.



OR

XI. Explain the laws of Boolean algebra.

XII. If a connected planar graph G has 'n' vertices, 'e' edges and 'r' regions, then prove that $n - e + r = 2$.

OR

XIII. Explain and prove Max-flow Min cut theorem.

XIV. Show that the set $\{0, 1, 2, 3, 4, 5, 6\}$ is a field with respect to addition modulo 7 and multiplication modulo 7 as the two compositions.

OR

XV. State and prove Fermat's little theorem.

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