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## MCA DEGREE I SEMESTER EXAMINATION NOVEMBER 2014

## CAS 2104 DISCRETE MATHEMATICAL STRUCTURES <br> (2010 Revision - Supplementary)

Time: 3 Hours
Maximum Marks: 50
PART A
(Answer $\boldsymbol{A} \boldsymbol{L} \boldsymbol{L}$ questions)
I. (a) Prove that $(A \cap B)^{\prime}=A^{\prime} \cup B^{\prime}$
(b) Prove that $p \rightarrow p v q$ is a tautology.
(c) Define a Lattice.
II. (a) State the principle of inclusion and exclusion.
(b) In how many ways can 4 cards be selected from a pack of cards so as to include atleast one diamond?
(c) If $A=\{1,2,3,4,5\}$ and $R=\{(1,1),(1,2),(2,3),(3,5),(3,4),(4,5)\}$ compute $R^{2}$ and $R^{\infty}$.
III. (a) Solve $a_{n}=4 a_{n-2}$.
(b) Find the characteristic function of reccurence relation $a_{n}+6 a_{n-1}+9 a_{n-2}=9$.
(c) Find an explicit formula for the sequence defined by $a_{n}=5 a_{n-1}-6 a_{n-2}$ with initial conditions $a_{1}=2$ and $a_{3}=1$.
IV. (a) Obtain the disjunctive normal form of $\left(x^{\prime} \wedge y\right) \vee(X \wedge z)$.
(b) Draw the circuit represented by $x y+\bar{x} y$.
(c) Prove that $a \wedge(a \vee b)=a$.
V. (a) Draw a finite automation that accept all strings of zeros and ones that starts with 111.
(b) Distinguish between deterministic and non-deterministic finite automata.
(c) Define regular language.

## PART B

VI. Prove by method of mathematical induction $1+2+2^{2}+\ldots+2^{n}=2^{n+1}-1$.

OR
VII. Convert the following argument into the language of symbols and check their validity. "Either the moon is cool or oxygen is a metal. The moon is cool. Therefore, oxygen is metal".
VIII. If $R=\{(1,4),(2,1),(2,2),(2,3),(3,2),(4,3),(4,5),(5,1)\}$ on the set $A=\{1,2,3,4,5\}$, then find $M_{R}, M_{R}{ }^{2}$ and $M_{R}{ }^{3}$.

OR
IX. State Pigeonhole principle. Find the minimum number of boys in a community to be sure that 5 of them are born in the same month.
X. Solve the recussence relation $a_{n}+5 a_{n-1}+6 a_{n-2}=3 n^{2}-2 n+1$.

OR
XI. Give an explicit formula for Fibonaci sequence and solve it.
XII. Using Karnaugh Map to minimize the Boolean expression $x y+x^{\prime} y+x y^{\prime}+x^{\prime} y^{\prime}$.

OR
XIII. Simplify the Boolean expression $x^{\prime} z+x^{\prime} y+x y^{\prime} z+y z$.
XIV. State and prove pumping Lemma.

OR
XV. Consider the finite state automaton $B$ defined by the following table.

| $\mathrm{S} \backslash \mathrm{A}$ | a | b | c |
| :--- | :--- | :--- | :--- |
| $\mathrm{S}_{0}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{0}$ | $\mathrm{~S}_{2}$ |
| $\mathrm{~S}_{1}$ | $\mathrm{~S}_{0}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{0}$ |
| $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{0}$ |
| $\mathrm{~S}_{3}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{0}$ | $\mathrm{~S}_{1}$ |

(i) What are the states of B?
(ii) Draw the transition diagram of B .

