MCA DEGREE I SEMESTER EXAMINATION DECEMBER 2015
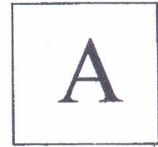
CAS 2101 COMBINATORICS AND GRAPH THEORY
(*Supplementary*)

Time: 3 Hours   Maximum Marks : 50

PART A
(Answer ALL questions)

$(15 \times 2 = 30)$

I.   (a)   Show that for each integer $n > 0$
            $$nC_0 + nC_1 + nC_2 + \ldots\ldots\ldots nC_n = 2^n$$

      (b)   Solve the recurrence relation $a_n - 3a_{n-1} = n$, $n \geq 1$, $a_0 = 1$.

      (c)   A committee of eight is to be formed from 16 men and 10 women. In how many
            ways can the committee be formed if
                (i)   there is no restrictions?
                (ii)   at least six men in the committee?

II.   (a)   Prove that an mxm matrix whose elements belong to $(-1, 1)$ is a Hadamad matrix
            if and only if its columns are orthogonal.

      (b)   Why is a square block design called symmetric?

      (c)   What is an error correcting code? Explain by giving an example.

III. (a)   Define subgraph and path in a graph with example.

      (b)   Explain traveling salesman problem.

      (c)   Define a fundamental cycle and fundamental cut-set in a graph with an example.

IV.   (a)   Show that a complete graph on four vertices is self dual.

      (b)   Draw the undirected graph whose incidence matrix is the following

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

      (c)   Prove that $\lambda^4 - 3\lambda^3 + 3\lambda^2 + 4$ cannot be a chromatic polynomial of any graph.

V.   (a)   Let $A = \{1, 2, 3, 4\}$ and the relation R on A is given by
            $$R = \{(1,1), (1,2), (2,1), (2,2), (2,4), (3,4), (4,1)\}.$$
            Draw the digraph of R.

      (b)   Define a tournament. Illustrate with an example that a tournament has a directed
            Hamiltonian path.

      (c)   Give one application of a rooted tree in the study of Computer Science.

PART B

<div align="right">$(5 \times 4 = 20)$</div>

VI.   In how many different ways can eight identical cookies be distributed among three distinct children if each child receives at least two cookies and not more than four cookies.

OR

VII.   What is a rook polynomial? Find the rook polynomial for the standard 8×8 chess board.

VIII.   Define a hamming code and hamming distance. For two code words define an addition '+' and multiplication '*' and show that if x and y are code words in a Hamming code C with weights $w(x)$ and $w(y)$ then

$$w(x+y) = w(x) + w(y) - zw(x*y)$$

OR

IX.   What is a t-error correcting code? Prove that a code C is t-error correcting if and only if $d(c) \geq zt + 1$

X.   Prove that a connected graph is Euler if and only if all the vertices are of even degree.

OR

XI.   Explain the Kruskal's algorithm for finding minimal spanning tree.

XII.   State and prove Kuratowiski's theorem.

OR

XIII.   State the five colour theorem. Write the chromatic polynomial and the chromatic number of the complete graph on four vertices K_4.

XIV.   Prove that there are n^{n-2} labeled trees with n vertices $(n \geq 2)$

OR

XV.   Explain tree searching. What is pre order and post order searching? Give example to illustrate.

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