MCA DEGREE I SEMESTER EXAMINATION DECEMBER 2013

CAS 2101 COMBINATORICS AND GRAPH THEORY

(Regular and Supplementary)

Time: 3 Hours　　　　Maximum Marks : 50

PART A

(Answer *ALL* questions)

(15 x 2 = 30)

I. (a) Show that there are $(n-1)!$ cyclic permutations of a set of 'n' points.

(b) How many combinations of 5 people can be chosen from 20 men and 12 women if
- (i) exactly 3 men must be on each committee
- (ii) at least 4 women must be on each committee

(c) Find the rook polynomial for an ordinary 3×3 board.

II. (a) Define a (v, k, λ) design with an example.

(b) What is Hadamard matrix?

(c) Explain Leech's lattice.

III. (a) Differentiate Euler graphs and Hamiltonian graphs.

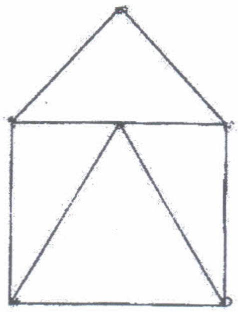
(b) Define isomorphism of two graphs.
Examine whether the following graphs are isomorphic.

(c) Define edge connectivity and vertex connectivity of a graph. Give examples.

IV. (a) What is meant by geometric dual of a graph?

(b) Define adjacency matrix and incidence matrix of a graph.

(c) Define chromatic number of a graph. Find the chromatic number of the following graph.



V. (a) Define a tournament. Give an example.

(b) What is condensation?

(c) Define arboresence. Give an example.

(P.T.O.)

PART B

(5 x 4 = 20)

VI. Use the method of generating functions to solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n$; $n \geq 2$ with initial conditions $a_0 = 2,\ a_1 = 8$

OR

VII. A survey of 500 television viewers produced the following information: 285 watch football games, 195 watch hocky games, 115 watch basketball games, 45 watch football and basketball games, 70 watch football and hockey games, 50 watch hockey and basketball games, and 50 do not watch any of the three kinds of games.

(i) How many people in the survey watch all three kinds of games?
(ii) How many people watch exactly one of the games?
(iii) How many people watch at least one of the games?

VIII. Let X be a Steiner system of order n and Y a subsystem of order m, where $m < n$, show that $n \geq 2m+1$. State the conditions for $n = 2m+1$.

OR

IX. Prove that each normalized Hadamard matrix A of order $4m \geq 8$ yields a $(4m-1,\ 2m-1,\ m-1)$ configuration.

X. Explain the properties of trees.

OR

XI. Prove that every circuit has an even number of edges in common with any cutset.

XII. State and prove Euler's formula, for planar graphs. Show that K_5 is non planar.

OR

XIII. (a) Prove that every tree with two or more vertices is 2-chromatic.
(b) Prove that a graph G with at least one edge is 2-chromatic if and only if it has no cycles of odd length.

XIV. Prove that every complete tournament has a directed Hamiltonian path.

OR

XV. State and prove Cayley's theorem.

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